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GRAVITY TECTONICS AND SEISMIC GAPS IN THE MANTLE

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ABSTRACT

The concept of gravity tectonics is applied to reveal the major clue as to the conditions which result in the correspondence of seismic and tectonic gaps in the mantle. An asymptotic theory is developed for the calculation of the thrust and moment when a descending lithospheric plate encounters resistance to its downward motion in the mesosphere. Dynamic analysis falls into two parts: (1) deriving equations for forces in the descending lithosphere, (2) deducing moment distribution which causes the detachment of lithosphere. For the analysis of forces a mathematical theory of shells is given. In order to determine the detachment mechanism, solutions of equations are obtained by asymptotic integration. It is found that a thrust N_ϕ coupled with a moment M_ϕ due to gravitational forces generated by density contrast may play a key role in the initial detachment of a piece of descending lithosphere. The results are in agreement with the observed seismic gaps beneath South America, Toga-Fiji, New Zealand and New Hebrides regions.

INTRODUCTION

Observations of the variations in attenuation and velocity of seismic waves beneath island arcs (Barazangi, et al., 1973; Isacks and Barazangi, 1973; Pascal, et al., 1973; Stauder, 1973) have revealed remarkable gaps in seismicity. These gaps are of interest because of the implication that portions of the lithosphere can break-off from the descending plate and exist as isolated slabs in the mantle. Although the existence of isolated lithospheric slabs is a natural explanation of much of the data on which seismic gaps are based, the dynamic aspects of detachment of these slabs are unknown. The present paper is devoted to studying the tectonic setting of seismic gaps beneath island arcs.

In this paper a model of gravity tectonics (Hales, A. L., 1969; Jacoby, 1972) is developed to explain conditions under which the correspondence of gaps in seismicity and tectonics may occur. Computation of the gravity effect of the downgoing lithosphere beneath island arcs (Elsasser, 1969; Isacks and Molnar, 1971) is complicated by its material properties. In order to obtain the first approximation, the density contrast between the descending lithosphere and surrounding asthenosphere is assumed to be uniform (Oxburgh and Turcotte, 1970) and the underthrusting plate is assumed to be elastic to maintain its shape (Sykes, 1966; McKenzie, 1969; Liu, 1973; Watts and Talwani, 1974). Analysis of the edge effect of thrust and moment (Love, 1944; Girkmann, 1956) due to gravitational body forces reveals a major clue for the initial detachment of descending lithosphere.

EQUATIONS OF THE PROBLEM

Several studies on the thermal regimes of downgoing slabs have been carried out using different models and techniques. (Turcotte and Oxburgh, 1968; McKenzie, 1969, 1970; Minear and Toksoz, 1970; Hasebe, et al., 1970; Griggs, 1972; Toksoz, et al., 1971, 1973) In addition to heat conduction from the surrounding mantle, the slab is heated by internal heat sources. These consist of radioacticity, phase changes, shear heating and adiabatic compression. Ideally, we would like to be able to specify the properties of the mantle and solve the time-dependent equations of deformation for the descending slabs, including the effects of temperature and pressure dependent parameters and internal heat sources. This task is analytically impossible and presents formidable problems numerically. Furthermore, it may well be that unless simpler models are first understood, complex cases would not be interpretable in terms of the contribution to the dynamics that each case provides. It seems necessary, and perhaps even desirable, to consider relatively simple model problems.

Assumptions and simplifications, common to all model problems, are required to obtain a tractable set of governing equations of detachment for the descending slabs in the mantle. We seek to generate models sufficiently simple to be solvable but which retain the main elements of the dynamics of the lithospheric detachment. One of the serious assumptions in tectonic plate theory is the rigidity of the plate. Its validity needs to be justified by the first order of approximation of the dynamical principle of deformable bodies. In this model, the descending slab is considered to be elastic, but the role of internal heat sources and phase boundaries are not considered. The temperature and pressure influences on the mechanical properties of the slab are not included within the scope of this paper. Linearized density in the slab and in the asthenosphere is assumed. The mesosphere, lying below 600 km depth, is assumed to be more dense than the asthenosphere, but no specific values for density are required. The assumed density contrast of about $0.05\,g\,cm^{-3}$ between the slab and its surrounding asthenosphere is the dominant feature of this model. These assumptions and simplifications, which impose limitations in representing the geophysical problem, lead to analytical solutions of the model that can be confirmed by observations of the tectonic plates.

Let us consider an element cut from the outer shell of the earth by two adjacent meridian planes and two sections perpendicular to the meridians (Fig. 1). In a axisymmetric shell the following resultant forces and moments per unit length occur: (1) in the cross-section ϕ = constant: N_ϕ, Q_ϕ and M_ϕ, (2) in the cross-section θ = constant: N_θ and M_θ. The stresses can be reduced to the resultant force $N_\theta r_1 d\phi$ and resultant moment $M_\theta r_1 d\phi$. The side of the element perpendicular to the meridians which is defined by the angle ϕ is acted upon by normal stresses which result in the force $N_\phi r_2 \text{Sin}\phi d\theta$ and the moment $M_\phi r_2 \text{Sin}\phi d\theta$ and by shearing stresses which reduce the force $Q_\phi r_2 \mathrm{Sin}\phi d\theta$ normal to the shell. The three equations of equilibrium are (Love, 1944; Girkmann, 1956)

$$\begin{aligned}
&\frac{d}{d\phi}(N_\phi r_o) - N_\theta r_1 \cos\phi - Q_\phi r_o = 0 \\
&N_\phi r_o + N_\theta r_1 \sin\phi + \frac{d}{d\phi}(Q_\phi r_o) = 0 \\
&\frac{d}{d\phi}(M_\phi r_o) - M_\theta r_1 \cos\phi - Q_\phi r_o r_1 = 0
\end{aligned} \tag{1}$$

in which the resultant forces N_θ, N_ϕ, shearing force Q_ϕ and resultant moments M_ϕ and M_θ are five unknown quantities. The number of unknowns can be reduced to three if we express N_θ, N_ϕ, M_ϕ and M_θ in terms of the components V and W of displacement (Fig. 2). The strain components of the middle surface of the element are

$$\epsilon_\phi = \frac{1}{r_1}\frac{dV}{d\phi} - \frac{W}{r_1} \qquad\qquad \epsilon_\theta = \frac{V}{r_2}\cot\phi - \frac{W}{r_2} \tag{2}$$

from which, by Hook's law, we obtain

$$\begin{aligned}
N_\phi &= \frac{Eh}{1-\nu^2}\left[\frac{1}{r_1}\left(\frac{dV}{d\phi} - W\right) + \frac{\nu}{r_2}(V\cot\phi - W)\right] \\
N_\theta &= \frac{Eh}{1-\nu^2}\left[\frac{1}{r_2}(V\cot\phi - W) + \frac{\nu}{r_1}\left(\frac{dV}{d\phi} - W\right)\right]
\end{aligned} \tag{3}$$

Where E is modulus of elasticity, h is the thickness of the shell and ν is Poisson's ratio.

To obtain similar expressions for M_θ and M_ϕ, we consider the changes of curvature of the shell element. Considering the upper and lower side of that element, the initial angle between these two sides is $d\phi$. Because of the displacement V along the meridian, the upper side of the element rotates with respect to the perpendicular to the meridian plane by the amount V/r_1. As a result of the displacement W, the same side further rotates about the same axis by the amount $dW/(r_1\,d\phi)$. Hence, the total rotation of the upper side of the element is $V/r_1 + dW/(r_1\,d\phi)$. For the lower side of the element the rotation is

$$\frac{V}{r_1} + \frac{dW}{r_1\,d\phi} + \frac{d}{d\phi}\left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right)d\phi$$

Therefore, the change of curvature of the meridian is

$$\lambda_\phi = \frac{1}{r_1}\,\frac{d}{d\phi}\left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right) \tag{4}$$

To find the change of curvature in the plane perpendicular to the meridian, we observe that the normal to the right lateral side of the element makes an angle $\pi/2 - \cos\phi\,d\theta$ with the tangent to the y-axis. Therefore, the rotation of the right side in its own plane has a component with respect to the y-axis equal to

$$-\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\;\cos\phi\,d\theta$$

This results in a change of curvature

$$\lambda_\theta = \left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right)\frac{\cot\phi}{r_2} \tag{5}$$

Using equations (4) and (5), we have

$$\begin{aligned} M_\phi &= -D(\lambda_\phi + \nu\lambda_\theta) \\ &= -D\left[\frac{1}{r_1}\,\frac{d}{d\phi}\left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right) + \frac{\nu}{r_2}\left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right)\cot\phi\right] \end{aligned} \tag{6}$$

$$M_\theta = -D(\lambda_\theta + \nu\lambda_\phi)$$

$$= -D\left[\left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right)\frac{\cot\phi}{r_2} + \frac{\nu}{r_1}\,\frac{d}{d\phi}\left(\frac{V}{r_1} + \frac{dW}{r_1\,d\phi}\right)\right] \qquad \text{(6 Cont'd.)}$$

where the flexural rigidity is

$$D = \frac{Eh^3}{12(1 - \nu^2)}$$

Substituting equations (3) and (6) into equation (1), we obtain three equations with three unknown quantities V, W and Q_ϕ. Simplification of these equations can be obtained by transformation to new variables.

TRANSFORMATION OF EQUATIONS

By using the third equation in (1) the shearing force Q_ϕ can be eliminated and the three equations reduced to two equations with the unknowns V and W. Considerable simplification of the equations can be obtained by introducing new variables. As the first of the new variables, we take the angle of rotation of a tangent to a meridian. We define this angle by

$$\Phi = \frac{V}{r_1} + \frac{dW}{r_1\,d\phi} \qquad (7)$$

As the second variable we take the quantity

$$U = Q_\phi\, r_2 \qquad (8)$$

To simplify the transformation of equations to the new variables, we may consider the forces in the portion of the shell above the parallel circle defined by the angle ϕ. They are governed by

$$r_o \sin\phi \, d\theta \, N_\phi + r_o \cos\phi \, d\theta \, Q_\phi = 0$$

from which

$$N_\phi = -\frac{\cot\phi}{r_2} U \tag{9}$$

Substituting equation (9) into the second of equations in (1), we find

$$N_\theta = -\frac{1}{r_1}\frac{dU}{d\phi} \tag{10}$$

Thus, N_ϕ and N_θ are both expressed in terms of U which is dependent on Q_ϕ as defined by equation (8).

To establish the first equation connecting Φ and U, we use equation (3) from which we obtain

$$\frac{dV}{d\phi} - W = \frac{r_1}{Eh}(N_\phi - \nu N_\theta) \tag{11}$$

$$V\cot\phi - W = \frac{r_2}{Eh}(N_\theta - \nu N_\phi) \tag{12}$$

By eliminating W from equations (11) and (12), the result is

$$\frac{dV}{d\phi} - V\cot\phi = \frac{1}{Eh}\left[(r_1 + \nu r_2)N_\phi - (r_2 + \nu r_1)N_\theta\right] \tag{13}$$

Differentiation of equation (13) with respect to ϕ gives

$$\cot\phi\frac{dV}{d\phi} - \frac{V}{\sin^2\phi} - \frac{dW}{d\phi} = \frac{d}{d\phi}\left[\frac{r_2}{Eh}(N_\theta - \nu N_\phi)\right] \tag{14}$$

By eliminating $dV/d\phi$ from equations (13) and (14), the result is

$$\begin{aligned} \Phi &= \frac{1}{r_1}\left(V + \frac{dW}{d\phi}\right) \\ &= \frac{\cot\phi}{Eh\, r_1}\left[(r_1 + \nu r_2)N_\phi - (r_2 + \nu r_1)N_\theta\right] \\ &\quad - \frac{1}{Eh\, r_1}\frac{d}{d\phi}\left[r_2\left(N_\theta - \nu N_\phi\right)\right] \end{aligned} \tag{15}$$

Substituting equations (9) and (10) in (15), we obtain the following equation relating to Φ and U.

$$\begin{aligned} &\frac{r_2}{{r_1}^2}\frac{d^2U}{d\phi^2} + \frac{1}{r_1}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\cot\phi - \frac{r_2}{r_1 h}\frac{dh}{d\phi}\right]\frac{dU}{d\phi} \\ &- \frac{1}{r_1}\left(\frac{r_1}{r_2}\cot^2\phi + \frac{\nu}{h}\frac{dh}{d\phi}\cot\phi - \nu\right)U = Eh\,\Phi \end{aligned} \tag{16}$$

The second equation relating to Φ and U is obtained by substituting equations (6), (7) and (8) in the third of the equations in (1). In this way we find

$$\begin{aligned} &\frac{r_2}{{r_1}^2}\frac{d^2U}{d\phi^2} + \frac{1}{r_1}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\cot\phi + 3\frac{r_2}{r_1 h}\frac{dh}{d\phi}\right]\frac{dU}{d\phi} \\ &- \frac{1}{r_1}\left(\frac{r_1}{r_2}\cot^2\phi - \frac{3\nu}{h}\frac{dh}{d\phi}\cot\phi + \nu\right)\Phi = -\frac{U}{D} \end{aligned} \tag{17}$$

Therefore, the problem of the membrane tectonics (Turcotte, 1974) of the underthrusting shell is reduced to the integration of equations (16) and (17). For the case of constant thickness, the terms containing $dh/d\phi$ as a factor vanish, and the derivatives of the unknowns Φ and U in both equations have the same coefficients. By introducing the notation

$$\mathcal{L}(X) = \left\{ \frac{r_2}{r_1^2} \frac{d^2}{d\phi^2} + \frac{1}{r_1} \left[\frac{d}{d\phi} \left(\frac{r_2}{r_1} \right) + \frac{r_2}{r_1} \cot \phi \right] \frac{d}{d\phi} - \frac{\cot^2 \phi}{r_2} \right\} (X) \tag{18}$$

equations (16) and (17) can be represented in the following simplified forms:

$$\mathcal{L}(U) + \frac{\nu}{r_1} U = Eh\,\Phi \tag{19}$$

$$\mathcal{L}(\Phi) - \frac{\nu}{r_1} \Phi = -\frac{U}{D} \tag{20}$$

Performing the operator $\mathcal{L}$ on equation (19) gives

$$\mathcal{L}\mathcal{L}(U) + \nu\, \mathcal{L}\left(\frac{U}{r_1} \right) = Eh\, \mathcal{L}(\Phi) \tag{21}$$

Substituting equation (19) into (20),

$$\mathcal{L}(\Phi) = \frac{\nu}{r_1} \Phi - \frac{U}{D} = \frac{\nu}{r_1\, Eh} \left[\mathcal{L}(U) + \frac{\nu}{r_1} U \right] - \frac{U}{D}$$

we obtain

$$\mathcal{L}\mathcal{L}(U) + \nu\, \mathcal{L}\left(\frac{U}{r_1} \right) - \frac{\nu}{r_1} \mathcal{L}(U) - \frac{\nu^2}{r_1^2} U = -\frac{Eh}{D} U \tag{22}$$

If the radius of curvature r_1 is constant, equation (22) reduces to

$$\mathcal{L}\mathcal{L}(U) + \left(\frac{Eh}{D} - \frac{\nu^2}{r_1^2} \right) U = 0 \tag{23}$$

ASYMPTOTIC EXPRESSION OF $Q\phi$

By introducing, instead of the shearing force $Q\phi$, the new variable

$$\tau = Q_\phi \sin^{1/2} \phi \tag{24}$$

equation (23) becomes

$$\frac{d^4 \tau}{d\phi^4} + C_2 \frac{d^2 \tau}{d\phi^2} + C_1 \frac{d\tau}{d\phi} + (4 \lambda^4 + C_0)\, \tau = 0 \tag{25}$$

in which

$$C_0 = -\frac{63}{16 \sin^4 \phi} + \frac{9}{8 \sin^2 \phi} + \frac{9}{10}$$

$$C_1 = \frac{3 \cos \phi}{\sin^3 \phi}$$

$$C_2 = -\frac{3}{2 \sin^2 \phi} + \frac{5}{2}$$

$$\lambda^4 = \frac{1 - \nu^2}{4} \left(\frac{12 a^2}{h^2} + 1 \right)$$

In the derivation of equation (25), $r_1 = a$ is assumed. For the case of underthrusting plate, $a \cong 10^3$ km and $h \cong 10^2$ km, a/h can be regarded as large. Therefore, the value of $4\lambda^4$ is very large in comparison with the coefficients C_0, C_1 and C_2, provided the angle ϕ is not small. Since we shall be interested in moments in the leading part of the underthrusting plates in the mesosphere where $\phi \cong \gamma$ and γ is not small, we can neglect the terms with the coefficients C_0, C_1 and C_2. In this way we obtain the equation

$$\frac{d^4 \tau}{d\phi^4} + 4 \lambda^4 \tau = 0 \tag{26}$$

The general solution of equation (26) together with equation (24) gives

$$Q_\phi = \sin^{1/2}\phi\,[e^{\lambda\phi}(k_1\cos\lambda\phi + k_2\sin\lambda\phi) + e^{-\lambda\phi}(k_3\cos\lambda\phi + k_4\sin\lambda\phi)] \tag{27}$$

where k_1, k_2, k_3 and k_4 are the constants of integration. They must be determined from the conditions at the leading part of the descending plate. Since the moments produced by forces on the edge of a shell decrease as the distance from the edge increases (Love, 1944; Girkmann, 1956), it is permissible to take only the first two terms in equation (27) and assume

$$Q_\phi = e^{\lambda\phi}\sin^{1/2}\phi\,(k_1\cos\lambda\phi + k_2\sin\lambda\phi) \tag{28}$$

Similar mathematical analysis shows that Φ has the same oscillatory character.

APPROXIMATE SOLUTIONS

As a basis of an approximate investigation of the bending of the underthrusting plates, we take equations (19) and (20). For this purpose, these equations can be written as follows:

$$\frac{d^2Q_\phi}{d\phi^2} + \cot\phi\,\frac{dQ_\phi}{d\phi} - (\cot^2\phi - \nu)\,Q_\phi = Eh\,\Phi \tag{29}$$

$$\frac{d^2\Phi}{d\phi^2} + \cot\phi\,\frac{d\Phi}{d\phi} - (\cot^2\phi + \nu)\,\Phi = -\frac{a^2Q_\phi}{D} \tag{30}$$

Q_ϕ and Φ have the same oscillatory character as shown in equation (28) and are damped out as the distance from the leading edge of the plate increases.

Since λ is large, the derivative of equation (28) is large in comparison with the function itself and the second derivative is large in comparison with the first. This indicates that a satisfactory approximation can be obtained by neglecting the terms containing the functions Q_ϕ and Φ and their first derivatives in the left-hand side of equations (29) and (30). Therefore, they can be replaced by the following simplified system of equations

$$\frac{d^2 Q_\phi}{d\phi^2} = Eh\,\Phi \tag{31}$$

$$\frac{d^2\Phi}{d\phi^2} = -\frac{a^2}{D}\,Q_\phi \tag{32}$$

By eliminating Φ from these equations, the result is

$$\frac{d^4 Q_\phi}{d\phi^4} + 4\beta^4\, Q_\phi = 0 \tag{33}$$

where

$$\beta^4 = 3\,(1-\nu^2)\,\frac{a^2}{h^2}$$

The solution of equation (33) for our problem is

$$Q_\phi = K_1\, e^{\beta\phi} \cos\beta\phi + K_2\, e^{\beta\phi} \sin\beta\phi \tag{34}$$

K_1 and K_2 are to be determined from conditions at the leading part of the plate. In discussing the edge conditions, it is advantageous to introduce the angle $\psi = \gamma - \phi$. Substituting $\gamma - \psi$ for ϕ in equation (34) and using the new constants K and α, we can represent equation (34) in the form

$$Q_\phi = K\, e^{-\beta\psi} \sin(\beta\psi + \alpha) \tag{35}$$

Now, employing equation (9), we find

$$N_\phi = -K \cot(\gamma - \psi)\, e^{-\beta\psi} \sin(\beta\psi + \alpha) \tag{36}$$

Substituting equation (35) in equation (31), we obtain the angle of rotation

$$\Phi = -\frac{2\beta^2}{Eh} K\, e^{-\beta\psi} \cos(\beta\psi + \alpha) \tag{37}$$

The moment M_ϕ can be determined by introducing equations (7) and (37) in (6). Neglecting the terms containing Φ in these equations for $r_1 = r_2 = a$, we find

$$\begin{aligned} M_\phi &= -\frac{D}{a}\frac{d\Phi}{d\phi} \\ &= \frac{a}{\sqrt{2}\,\beta} K\, e^{-\beta\psi} \sin\left(\beta\psi + \alpha + \frac{\pi}{4}\right) \end{aligned} \tag{38}$$

With the aid of equations (36), (37) and (38), the model of gravity tectonics for the detachment of descending lithosphere can readily be treated.

CALCULATIONS

In order to calculate N_ϕ and M_ϕ in the underthrusting plates, we must determine the constants K and α in equations (36) and (38). This can be done by applications of force analysis (Fig. 3). According to the suggestion made by Elsasser (1969) and Isacks and Molnar (1971), the lithospheric plate is sinking under island arcs and exerting a downward gravitational pull. When the sinking lithosphere reaches the more dense, stronger methosphere region below the asthenosphere, downdip compression is evidenced in the earthquake patterns (Isacks and Molnar, 1971). This suggestion requires a positive density contrast $\Delta\rho$ of the sinking plate which is about 0.02 to 0.06 g cm^{-3} as estimated by Jacoby (1970) and by Oxburgh and Turcotte (1970). Such an estimation is supported by the observed gravity anomalies (Hatherton, 1969). Satellite gravity (Kaula, 1972) has the resolution to show the positive anomalies related to the oceanic trenches. To estimate the buoyant force on the plate, we consider the case of a plate with edge thrusting in the mesosphere and subject to the action of gravity. The vertical force is $W = A\Delta\rho g \cong (\ell h/\text{Sin}\gamma)\,\Delta\rho g$, where A is the volume per unit length along the strike, ℓ is the depth of the leading edge of the descending lithosphere, h is its thickness and γ is the dip of the plate measured from horizontal. The force F represents a push from the ridge due to the elevation of the ridge associated with ascending convection beneath the ridge and a horizontal traction on the base of the lithosphere due to convection in the upper mantle. If the descending plate is uncoupled from the horizontal lithosphere by vertical faults, no tensile force could be exerted directly on the horizontal plate from the descending one. If the tension is compensated by compression from the ridge (Jacoby, 1970), the component $S = W\text{Sin}\gamma$ parallel to the inclined plate may be in equilibrium with a vertical supporting force G and a horizontal thrust H in the mesosphere. This estimate is not yet complete. When the plate continues to dip into the more viscous and denser mesosphere due to force F, the resistance is likely to grow rapidly, and the leading edge of the plate encounters a resistance $(N_\phi)_{\phi = \gamma}$ which may be greater than S. Hence, for this case, we have two boundary conditions

$$\begin{aligned} (N_\phi)_{\phi = \gamma} &\geqslant \ell h \Delta\rho g \\ (M_\phi)_{\phi = \gamma} &= 0 \end{aligned} \tag{39}$$

By substituting $\psi = 0$ in equation (38) it can be seen that the second boundary condition in equation (39) is satisfied by taking the constant $\alpha = -\pi/4$. To satisfy the first boundary condition, we use equation (36) which gives

$$K \geqslant \frac{\ell h \Delta\rho g}{\sin(\pi/4)\cot\gamma}$$

Substituting the values of the constant K and α in equation (38), the result is

$$M_\phi \geqslant \frac{a}{\beta}\,\frac{\ell h \Delta\rho g}{\cot\gamma}\, e^{-\beta\psi} \sin(\beta\psi) \tag{40}$$

For ℓ = 600 km, h = 100 km, $\Delta\rho$ = 0.05 g cm^{-3}, g = 10^3 cm sec^{-2}, a = 1200 km, and $\gamma = 45°$, $\beta = 4.45$, we obtain

$$(N_\phi)_{\phi = \gamma} \geqslant 3.5 \times 10^{16} \text{ dyn cm}^{-1} \tag{41}$$

$$(M_\phi)_{\phi = \gamma} \geqslant 8.4 \times 10^{23}\, e^{-\beta(\gamma - \phi)} \sin(\gamma - \phi) \text{ dyn - cm cm}^{-1} \tag{42}$$

The corresponding values of M_ϕ from equation (42) are shown in Fig (4). In Fig (4) the maximum moment of body forces due to gravity generated by density contrast occurs at $\phi = 36°$. This implies that the thrust $(N_\phi)_{\phi = \gamma}$ coupled with the bending moment M_ϕ may provide a mechanism for the initial detachment of the descending lithosphere plate at a depth of about 460 km. Under the influence of the bending moment, M_ϕ curve represents the curvature of the plate. This analytical result reveals that the lithosphere breaks up but does not sink vertically along en echelon vertical fault as shown in Figure 4. The earthquake distribution beneath the New Hebrides arc as obtained by Dubois (1971) is illustrated in Figure 5. The horizontal extent of the deep earthquakes in Figure 5 suggest that the detachment of the slab is controlled by the gravitational bending moment due to density contrast. Therefore, the gravitational bending moment, M_ϕ, may provide an explanation for the similarity among the seismic and tectonic gaps beneath the South American, Tonga-Fiji, New Zealand and New Hebrides Island arcs as observed by Isacks and Barazangi (1973), Stauder (1973), Pascal, et al. (1973), Barazangi, et al. (1973) and Isacks and Molnar (1971). If this similarity is not fortuitous, the analytical solutions of this model are probably relevant to the geophysical problem. Previous studies (Toksoz, et al., 1973; Liu, 1973) lack the understanding of the detachment mechanism. This geophysical phenomenon is, however, not unexpected if the elastic model of gravity tectonics developed in this paper is accepted.

CONCLUSION

Seismic observations provide important information of the deep-seated gaps along converging plate boundaries. These gaps must be examined with studies of the tectonic plate deformation to determine whether the results of seismic observations are consistent with simple tectonic plate models. In this paper a mathematical model of gravity tectonics to explain and predict the seismic gaps in the mantle is developed. The results of the analysis are formulated by equation (36), (37) and (38). Numerical calculations lead to equation (42) and Figure 4. It is shown that the thrust coupled with a moment of body forces due to gravity on the leading portion of the underthrusting plate may play a key role in the initial detachment of a piece of descending lithosphere. Furthermore, the tectonic setting of seismic gaps presented in this paper reveals the clue as to the conditions under which they occur and the regional motions, forces and moments which must be responsible for their occurrence.

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FIGURE CAPTIONS

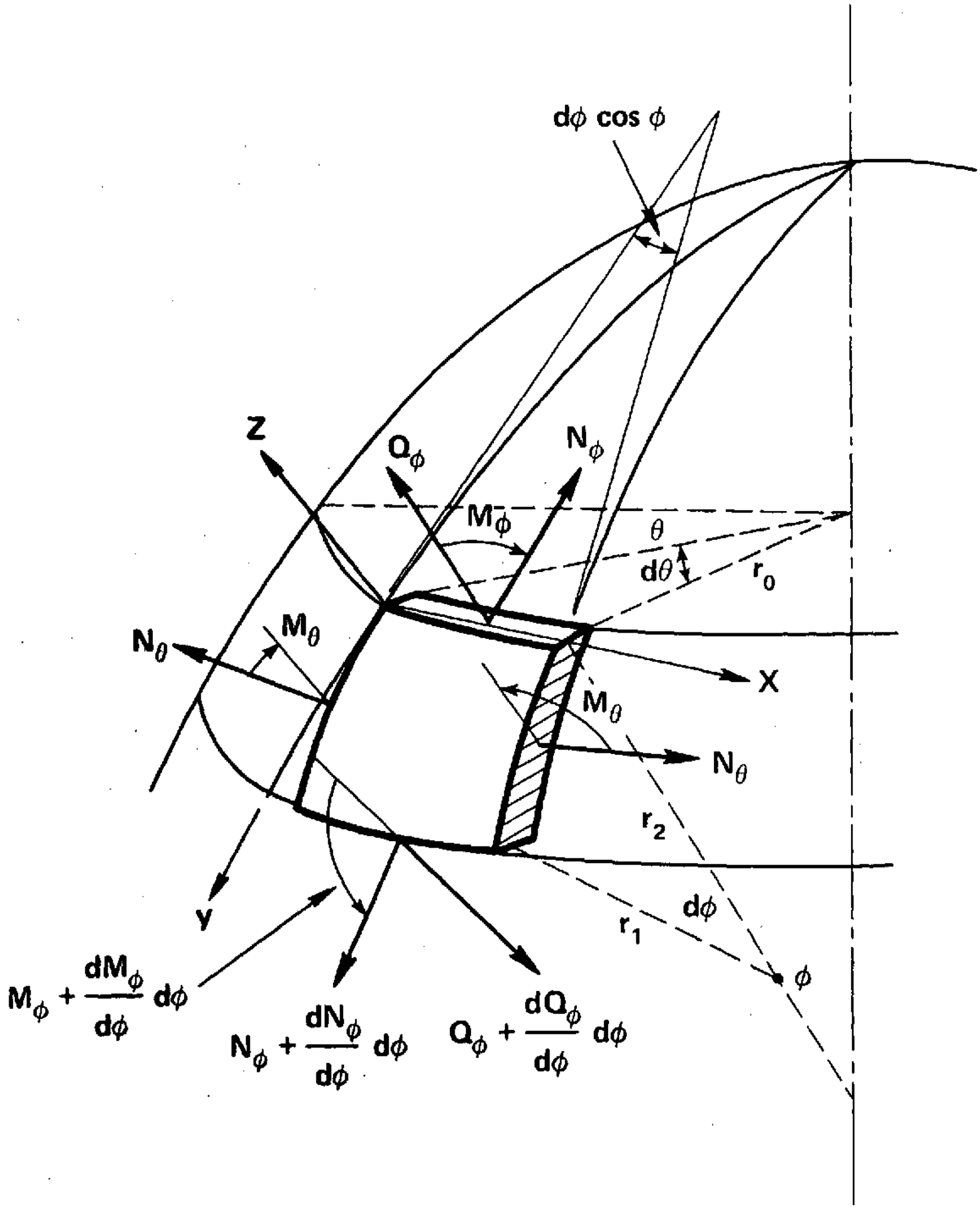
Figure 1. Forces in a Plate Element

Figure 2. Displacements in Plate

Figure 3. Force Diagram

Figure 4. Distribution of Moment M_ϕ Due to Gravitational Forces

Figure 5. Vertical Cross Section Perpendicular to the New Hebrides Arc Showing Earthquake Distribution Beneath the Arc and in the Detached Slab



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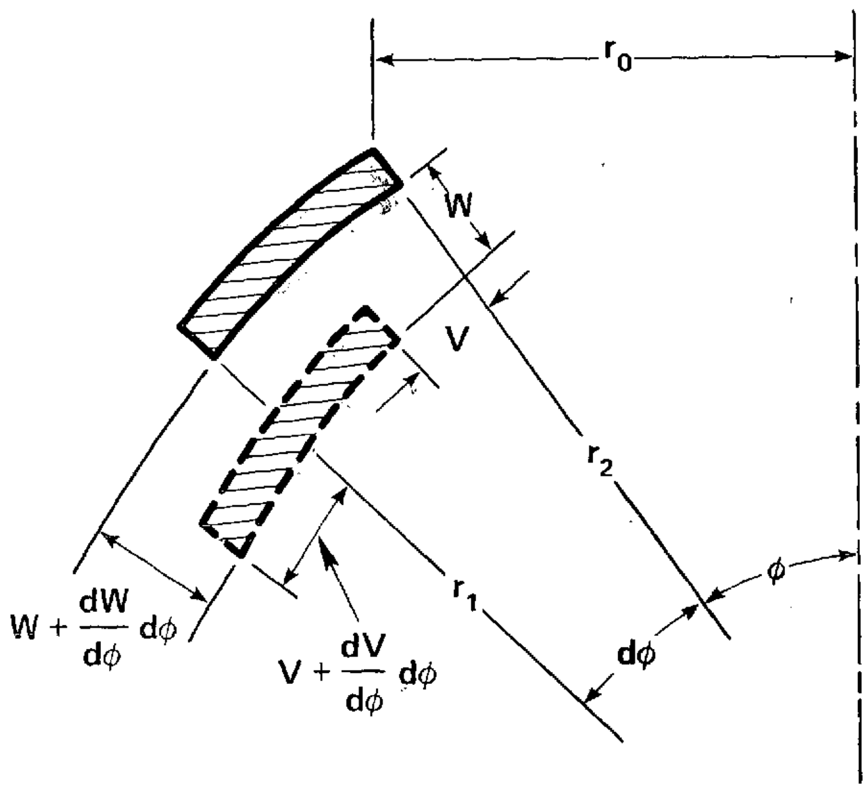
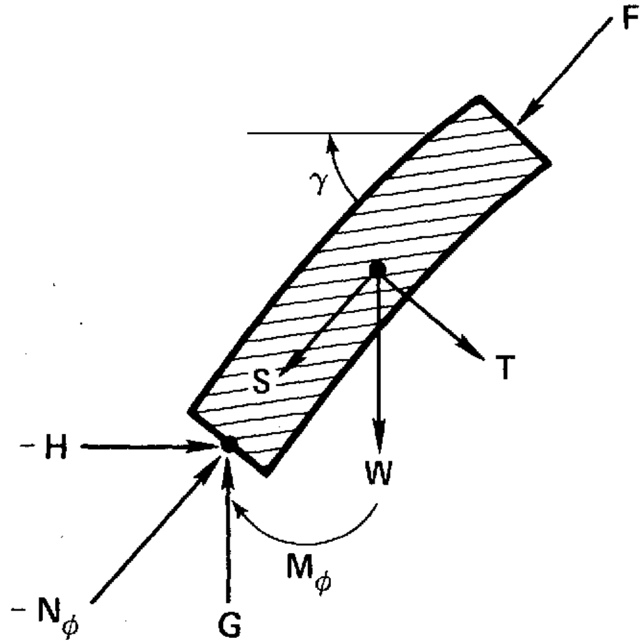


Figure 2. Displacements in Plate



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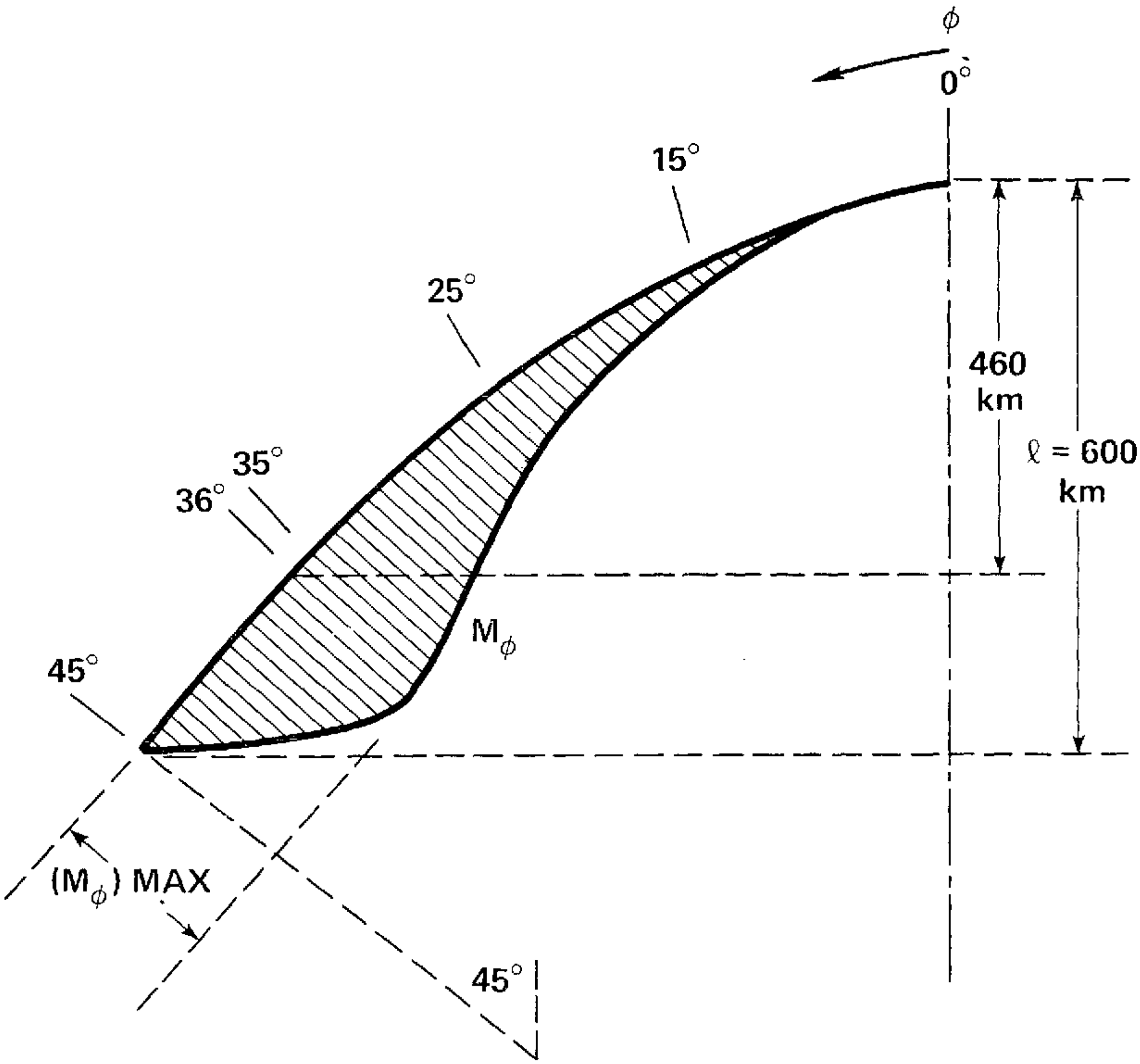

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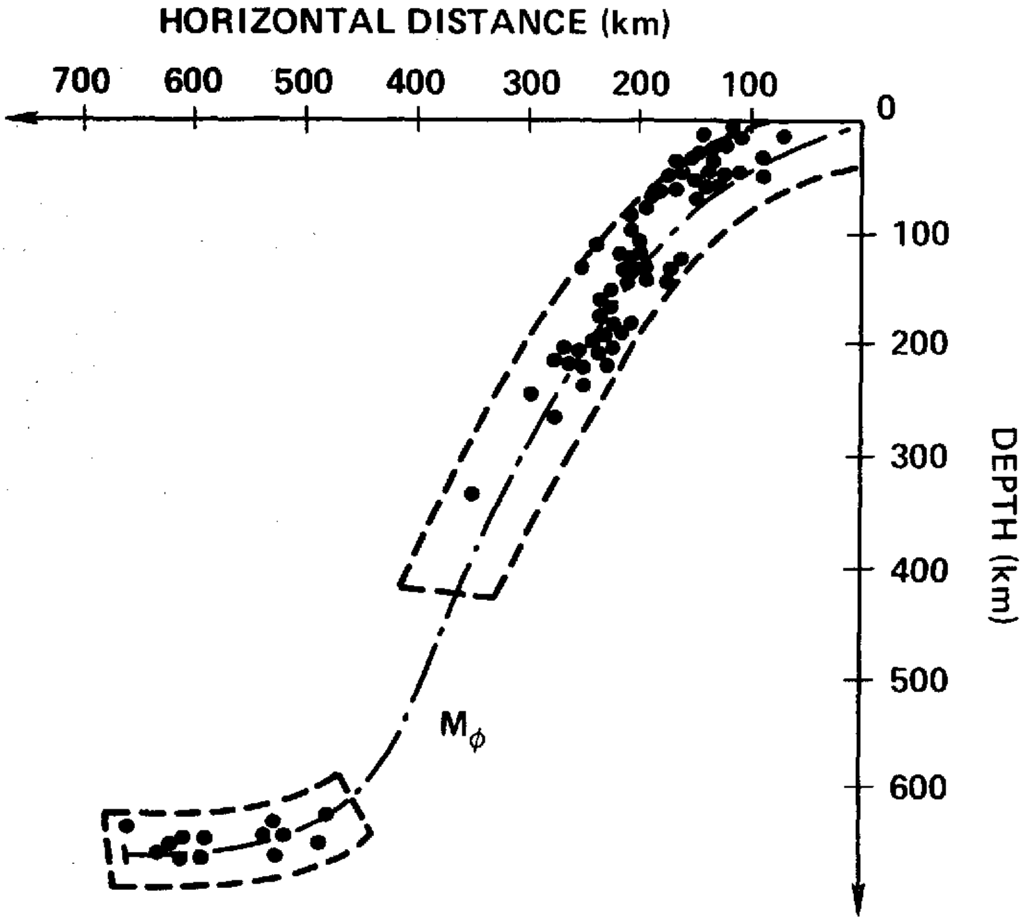

Figure 5. Vertical Cross Section Perpendicular to the New Hebrides Arc Showing Earthquake Distribution Beneath the Arc and in the Detached Slab